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INCLUDING PRESTRESSED CONCRETE

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INDEX OF ADVERTISERS

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Volume LIV, No. 7. London, July, 1959.

EDITORIAL NOTES

Electronic Calculating Machines.

Aids in making structural calculations have in the past been generally restricted to the use of a slide-rule, but a more versatile and valuable tool is now available in the form of electronically-operated computors which can perform not only the arithmetical processes but also some purely mathematical processes. Solutions of many complex problems can therefore be obtained more easily and quickly than hitherto, and it is claimed that some problems can be dealt with which up to now have not been practicable because of the great amount of time they require. At present the new aids are expensive, but some of their possibilities and the co-operation among engineers and professional bodies to reduce the cost were described at a Symposium on Electronic Digital Computors organised by the Cement and Concrete Association and held in London in May last.

The two principal types of electronic calculating machines are the analogue computor and the digital computor, the primary difference being that a number is represented in the former by a difference in voltage and in the latter by electrical impulses. An analogue computor, which works by balancing electrical forces, is particularly suitable when the information varies during the period of operation. The digital computor operates on the principle of counting the impulses corresponding to binary numbers into which ordinary decimal numbers are converted. When the information has been set in the digital computor it cannot be varied while the calculation is in progress; this type of computor is more convenient for the solution of structural problems.

Electronic computors can be used in structural engineering either to calculate the stresses in a structure whose dimensions and properties have been decided, or to produce the best design complying with specified limiting requirements. Although computors have been frequently used to calculate stresses, their application to the design of structures is less extensive and, in the case of redundant structures, is at present difficult. In all cases the machine must receive the relative data and explicit instructions on what is required to be done with the data. This information is recorded on punched tapes or cards or on magnetic tapes which are put into the computor, and the machine carries out these instructions until a solution is produced or until an item of information which is about to be used is incompatible with the preceding information, in which case the computor stops. The stage at which the interruption occurs can be traced and a correction made.

The procedure when using a computor is first to prepare the data, called a programme, and to decide the procedure for dealing with the data. This information is recorded on the tape or card, which is put into the machine. The result calculated by the machine is transcribed by the person who prepared the programme into a form suitable for use by the designer. Typical information for the analysis of a structure is the number and dimensions of the members, the position of the joints and members, the forces applied and the analysis required. The results which the machine provides are the stresses in, and deflections of, each member and the rotations of the joints.

A programme may be prepared by the owners of the computor or by a specially-trained engineer associated with the designer. The time taken by the machine to make a calculation may be as short as twenty-five seconds for the calculation of the stresses in a cylindrical vault subjected to symmetrical loading or forty-five seconds if the load is unsymmetrical; these periods are insignificant compared with the time required to prepare the programme, to put it into the machine, and to transcribe the results. The preparation of the programme is the longest task. To reduce the time needed to prepare a programme, short instructions for a computor are available, for example, for the calculation of moments of inertia and the differentiation of a mathematical function. Several programmes already exist, including the analysis of arches, cylindrical " shells ", planar frames with fixed or hinged ends, skew slabs, flat slabs and plates. Programmes are in preparation at Southampton University for dealing with helical stairs, the oscillation of tall chimneys, multiple-lamini in series, doubly-curved roofs, and the restricted flow of water through soils; the Cement and Concrete Association is collecting a library for all known programmes for structural and civil engineering problems. The computors at the Universities of Southampton and Manchester and at Northampton College of Technology are available for use by engineers.

Although a programme can be prepared for a computor to perform any of the mathematical operations used in engineering, a problem can be conveniently presented to the machine in the form of a matrix by means of which it can solve many simultaneous equations with great rapidity; consideration of the solution of structural problems by such means was given by Mr. A. H. Douglas in this journal for April and May last. The cost of the use of an electronic digital computor is from £20 to £75 per hour and the cost of preparing a programme is about 30 shillings per hour. Nevertheless it is said to be not only quicker but cheaper to solve more than six simultaneous equations by means of a computor than by ordinary methods.

Although a computor is a versatile calculating machine, it has no " intelligence "; for example, it cannot make a choice in the course of a calculation, so that exact instructions have to be provided for such an occurrence. Therefore, although it may be possible for a computor to select the most suitable design, instructions must be given on how to make the selection. Machines may eventually be used for making complex calculations upon which many engineers spend much time, with the result that engineers will have more time to devote to the planning that is so important in the preparation of a successful structure. It is, however, unlikely that machines will for many years, if ever, take the place of slide-rules, tables, and other aids to common calculations.

The Fire-Resistance of Prestressed Concrete.

By PROFESSOR THOMAS KLUZ (WARSAW).

THE writer recently had occasion to investigate the fire resistance of some proposed floors and roofs to be prestressed with pre-tensioned steel. The constituents of 1 cu. yd. of concrete were 2400 lb. of basalt, 1610 lb. of sand, and 1010 lb. of Portland cement ; the water–cement ratio was 0·33. The objects of the investigation were (1) to study the changes in the elements during heating ; (2) to determine the modulus of elasticity and the ultimate strength before and after heating ; and (3) to determine the losses of prestressing force caused by heating. Tests were made on (1) Twelve specimens 2 in. by 1·2 in. in cross section by 10 in. long for axial loading ; (2) Eighteen specimens 2 in. by 1·2 in. by 6 ft. 6 in., subjected to a temperature of 600 deg. C. on one side for a period of four hours ; (3) Twelve specimens 6 in. by 2 in. by 6 ft. 6 in. (*Fig.* 1) each composed of a 2-in. by 1·2-in. prestressed plank and concrete cast in place ; and (4) Six columns 4½ in. square and 2 ft. high. The results of the tests of specimens (1) to (3) are discussed in this article.

The soffit only of each specimen was heated by gas ; the other sides were protected by asbestos covers. The temperature was measured by a thermoelectric pyrometer, and thermocouples consisting of steel and constantan were concreted in the specimens ; with this equipment the temperature could be measured with an accuracy of 6 deg. C.

Compressive Tests.

Of the twelve specimens tested for axial compression, nine were prestressed and three were not ; three prestressed and two unstressed specimens were used as controls, and the remaining seven were heated and then tested. *Fig.* 2 shows the prestressed specimens Nos. 1, 2, and 3, the properties of which were determined without heating, and *Fig.* 3 shows the prestressed specimens Nos. 4 to 9 which were subjected to heat. *Fig.* 4 shows the unstressed specimens, of which Nos. 10

Fig. 1.—Composite Prestressed Beam.

and 12 were not heated. The specimens were made of concrete with a compressive strength of 5700 lb. per square inch, and were stressed by ten 1·5-mm. wires. The tensile stress σ_{ss} in the steel, after losses had occurred, was 176,200 lb. per square inch and the prestress σ_{bc} in the concrete was 2130 lb. per square inch.

During the tests the surface temperatures of the specimens were raised to 600 deg. C. during a period of 30 to 35 minutes, and kept constant for four hours. Some specimens were cooled in one hour, and others in a few minutes. The temperatures at the wires and on both sides of the specimens were recorded at intervals of five minutes. *Fig.* 5 shows the temperature diagram for specimen No. 6. All the specimens were tested in direct compression after a few days.

Fig. 2.—Prestressed Control Specimens.

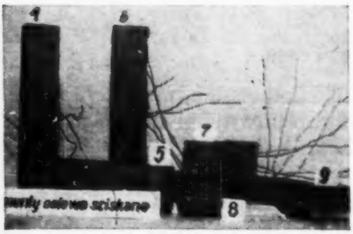


Fig. 3.—Prestressed Test Specimens.

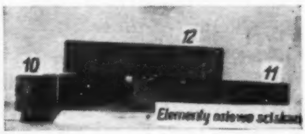


Fig. 4.—Unstressed Specimens.

CHANGES OF PRESTRESSING FORCE.—Splitting occurred at most of the wires as the temperature increased from 200 to 500 deg. C. No cracking or spalling of the cover of concrete was observed during slow or rapid cooling. The cooled specimens gave a metallic sound when struck; their colour became lighter than that of the control specimens.

After four days the shortening of the wires in relation to the length of the specimen was measured. The shortening was not constant for any specimen and varied from 0·059 mm. to 0·778 mm. in a gauge-length of 25 cm. The losses of prestress are approximately given by $\Delta\sigma_{zt} = \varepsilon_t . E_z$, in which E_z is 29,200,000 lb. per square inch and ε_t is the strain of the wires. The values of ε_t, $\Delta\sigma_{zt}$, σ_{zt}, and the greatest temperatures of the wires during heating, are given in *Table* I. The loss of prestressing force is seen to depend on the temperature to which the wires are heated. The method of cooling also probably influenced the losses (compare specimens 7 and 9 with specimen 8).

COMPRESSIVE STRENGTHS OF HEATED SPECIMENS.—*Table* II gives the compressive strengths of all the specimens. It should be noted that while the strength of the plain specimen 11 was about 39 per cent. less than that of the control specimens the strength of the comparable prestressed specimens 4 and 5 was reduced only by about 12·5 per cent. Comparison of the strengths of specimens 4, 5, and 6 indicates that the method of cooling greatly influences the loss of strength; with rapid cooling the loss is three times that with slow cooling.

TABLE I.—LOSSES OF PRESTRESSING FORCE CAUSED BY HEATING.

Test	Shortening of wires ε_t	Losses of prestressing forces due to heating $\Delta\sigma_{st}$ lb. per sq. in.	Stresses in wires after heating σ_{st} lb. per sq. in.	Temperature of wires during heating (deg. C.)
1. Specimens 4 and 5. Wires heated from one side. Slow cooling .	0·000618	18,050	158,150	372
2. Specimen 6. Wires heated from one side. Rapid cooling . . .	0·001420	41,400	134,800	472
3. Specimens 7 and 9. Wires heated from the upper side and the soffit. Slow cooling	0·001575	45,900	130,300	550
4. Specimen 8. Heating as in (3). Rapid cooling	0·002040	59,600	116,600	605

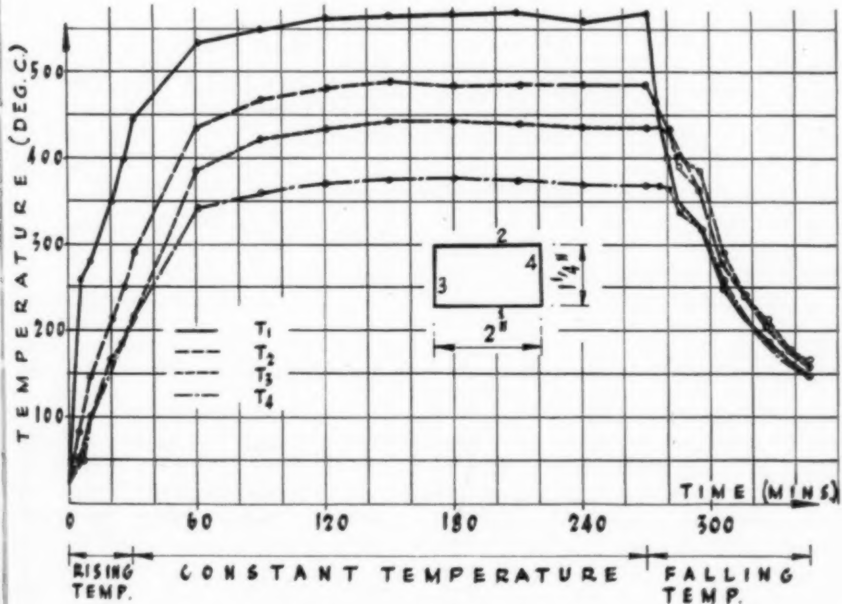


Fig. 5.—Temperature-Time Curves.

TABLE II.—COMPRESSIVE STRENGTHS OF SPECIMENS.

No. of test	No. of specimen	Mean temperature (deg. C.)			Method of heating	Method of cooling	Stress at failure		Remarks
		Soffit	Wires	Upper surface			lb. per sq. in.	%	
1	10 and 12	—	—	—	—	—	5970	100·0	Unstressed. Not heated.
2	11	599	—	374	one side	slow	3630	60·7	Unstressed. Heated.
3	1, 2 and 3	—	—	—	—	—	6880	100·0	Prestressed. Not heated.
4	4 and 5	597	366	331	one side	slow	6030	87·5	Prestressed. Heated.
5	6	596	449	291	,,	rapid	4130	60·2	
6	7 and 9	591	494	595	two sides	slow	3770	54·8	
7	8	588	578	603	,,	rapid	4480	65·2	

TABLE III.—MODULI OF ELASTICITY.

No. of test	No. of specimen	Temperature of wires or centre of specimen (deg. C.)	Method of heating	Method of cooling	Modulus of elasticity E_1		Remarks
					Tons per sq. in.	%	
1	10 and 12	—	—	—	3270	100·0	Unstressed. Not heated.
2	11	374	one side	slow	901	27·5	Unstressed. Heated.
3	1, 2 and 3	—	—	—	3430	100·0	Prestressed. Unheated.
4	4 and 5	366	one side	slow	1975	57·7	Prestressed. Heated.
5	6	499	one side	rapid	1945	56·8	Prestressed. Heated.
6	7 and 9	493	two sides	slow	1322	38·7	Prestressed. Heated.
7	8	578	two sides	rapid	1830	53·5	Prestressed. Heated.

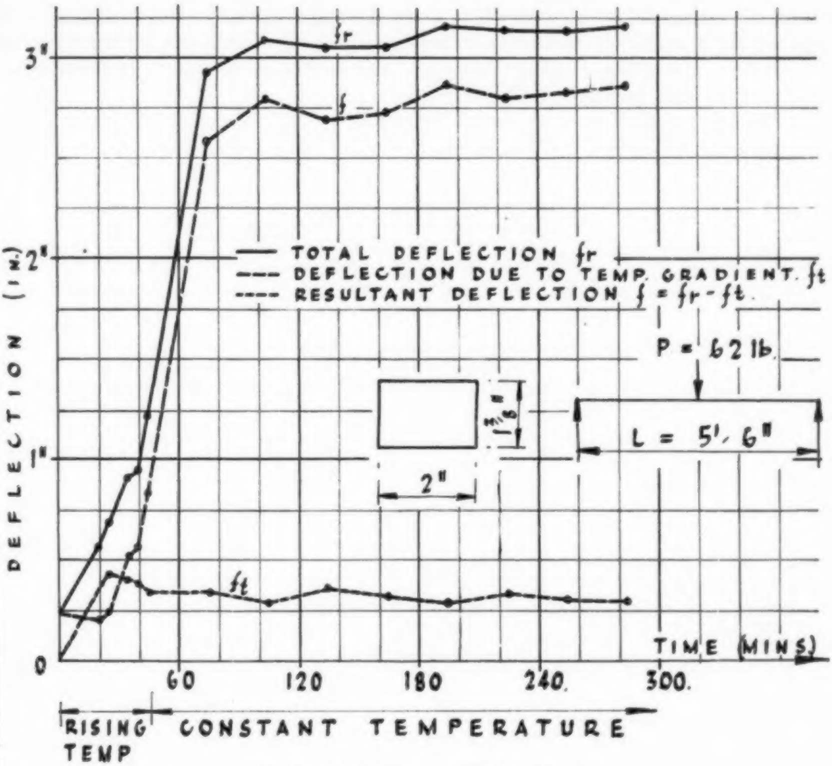


Fig. 6.—Deflection of Beam No. 2.

Moduli of Elasticity.—The modulus of elasticity of each specimen was investigated, and the results are given in *Table* III. The value of the modulus of the unstressed specimen decreased by about 72 per cent. and that of the corresponding prestressed specimens by about 42 per cent. In this respect, therefore, the prestressed specimens behaved better than the unstressed specimens.

Prestressed Beams with Pre-tensioned Steel.

The overall length of each specimen was about 6 ft. 6 in., and the span between supports was about 5 ft. 6 in. Twelve beams (1 to 12) were tested, and three (16 to 18) served as controls. Beams 1 to 3 supported a central load of 55 lb., the depth of the beams being 1·2 in. Nos. 4 to 6 and 10 to 12 were unloaded, and 7 to 9 supported a central load of 97 lb., the depth of the beams being 2 in. All beams except the controls were heated by a mixture of coal-gas and air to 500 deg. C. and maintained at that temperature for four hours in the furnace shown in *Fig.* 8. Specimens 5, 8, and 11 were rapidly cooled by water ; the remainder were allowed to cool slowly in air. During the tests the deflections were measured

by three strain-gauges ; a few weeks later the beams were tested to destruction. *Figs.* 6 and 7 show the deflections of beams 2 and 7. These measurements were required for the indirect determination of the moduli of elasticity of prestressed concrete at high temperatures, as follows.

The total deflections f_r shown in *Figs.* 6 and 7 comprise the deflections f_t due to differences of temperature between the upper and lower parts of the beams,

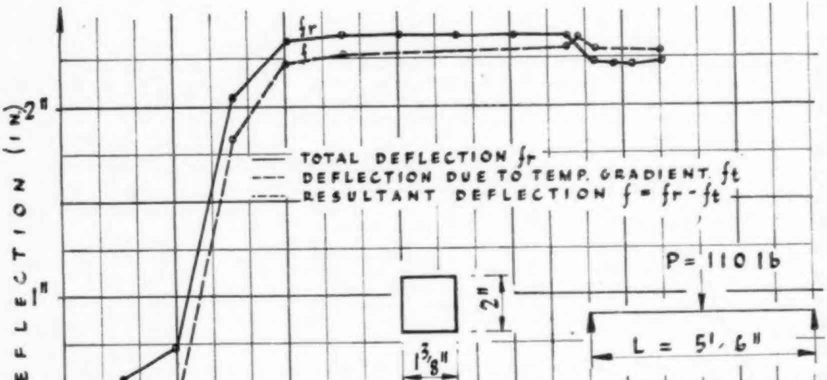


Fig. 7.—Deflection of Beam No. 7.

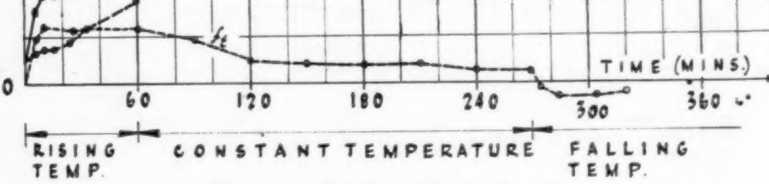


Fig. 8.—Prestressed Beam During Test.

and the additional deflections f which occurred during the tests as a consequence of the significant decrease in the modulus of elasticity which occurs at high temperatures.

Hence
$$f_r = f_t + f \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The bending moment M_t due to a difference in temperature of Δ_t between the top and bottom of a member is

$$M_t = \frac{2\alpha \Delta t EI}{h} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which α is the coefficient of linear expansion and h is the depth of the member. The deflection f_t caused by moments M_t acting at each end of the member is

$$f_t = \frac{M_t . l^2}{8EI} = \frac{\alpha \Delta t l^2}{4h} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

On this basis the deflections of beams 2 and 7 due to differences of temperature were determined assuming α for concrete made with basalt to be 0·000012. The difference between f_r and f_t is the deflection due to the external forces :

$$f = f_r - f_t \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The modulus of elasticity was then determined from

$$E_t = \frac{Pl^3}{48I} \cdot \frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the case of beam 2, with a range of temperature of 456 deg. C. to 511 deg. C., E_t was between 341,000 lb. and 370,000 lb. per square inch ; for beam 7, with a range of temperature of 330 deg. C. to 480 deg. C., E_t was between 341,000 lb. and 441,000 lb. per square inch.

Similar values were obtained for all the specimens. On this basis the following conclusions may be stated. (1) The modulus of elasticity of concrete at high temperatures decreases significantly from that at normal temperatures. This phenomenon may be compared with the increases of the value at low temperatures.[1] [2] (2) At a temperature of about 500 deg. C. the modulus of elasticity is less than one-twentieth of that at 20 deg. C. (*Table* IV). (3) After heating to a high temperature the value of the modulus of elasticity of prestressed concrete increases rapidly with cooling ; this has the effect of decreasing the deflections of prestressed beams, but the original value is never regained. The value of E some weeks after heating (determined at 20 deg.) is only about 60 per cent. of the original modulus.

TABLE IV.—VALUES OF MODULUS OF ELASTICITY OF PRESTRESSED CONCRETE AT HIGH AND LOW TEMPERATURES.

	Temperature (deg. C.)				Residual modulus at +20 deg. C.	
	+20	+500	+20	−40		
Modulus (lb. per sq. in.)	7,670,000	356,000	5,720,000	6,400,000	4,320,000	5,970,000
Remarks	High temperature		Low temperature		After heating	After cooling

Composite Prestressed Beams.

Beams 4 to 12 were tested ; Nos. 1 to 3 served as controls. *Fig.* 8 shows one of the beams during the test. The heat was applied to one side only ; beams 4, 7, 9, 11, and 12 were kept at a temperature of 500 deg. C. for four hours, and beams 5, 6, and 10 were kept at about 550 deg. C. Beams 4 to 6 were unloaded, and beams 7 and 9 to 12 supported a central load of 220 lb. The temperatures were measured at the upper and lower surfaces of the prestressed plank, at one of the prestressing wires, and at the upper surface of the beam. Beams 4, 6, 9, and 12 were slowly cooled in air ; beams 5, 7, 10, and 11 were rapidly cooled with water. *Fig.* 9 shows one of the beams at the start of a test ; *Fig.* 10 shows a tested beam. Some months after the tests the beams were loaded to destruction, measurements being taken of deflections and deformations.

Deformations were measured by six Hugenberger extensometers at the middle of the beam and two extensometers near the supports. The central extensometers were at one-third and two-thirds of the depth, on both sides, and on the upper face.

No fissures, cracks, slipping of wires, or damage to concrete were observed in the prestressed planks during the tests. During cooling, cracks appeared above

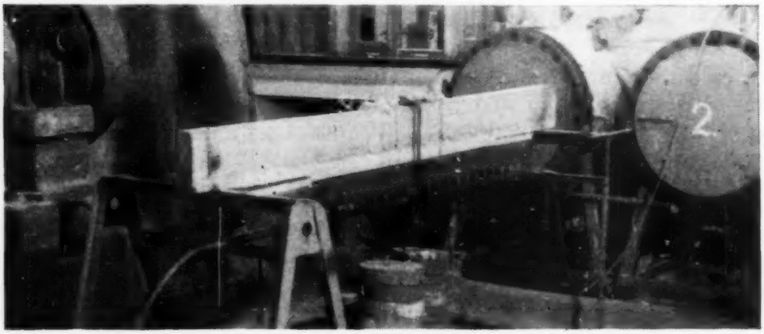


Fig. 9.—Composite Beam During Test.

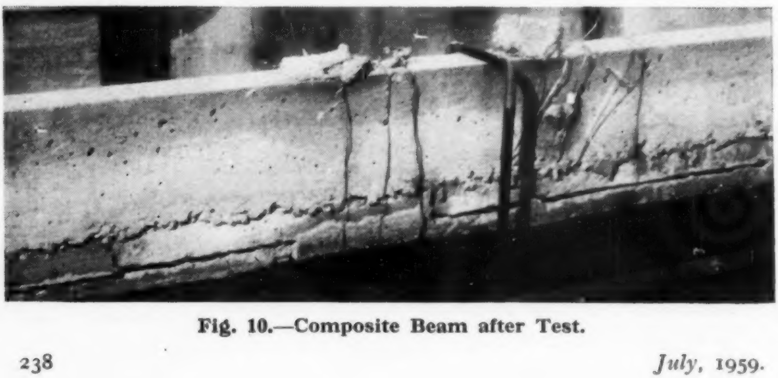


Fig. 10.—Composite Beam after Test.

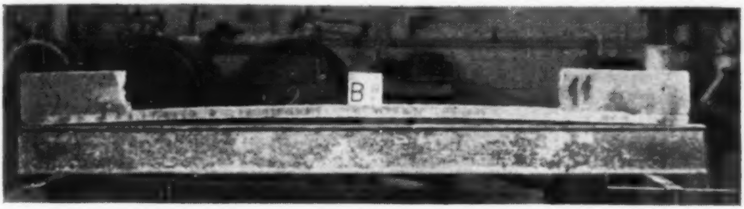


Fig. 11.—Beam Damaged during Cooling.

the prestressed planks near the supports, caused probably by differences of temperature, as the ends of the beams were not heated for a length of about 6 in. On some of the beams a cover of mortar about 0·6 in. thick, made with gypsum or cement and lime, was used; this fell away during the test. Only one beam (11) was damaged during rapid cooling (*Fig.* 11).

The results obtained are shown in *Table* V.

TABLE V.—ULTIMATE STRENGTH OF COMPOSITE BEAMS.

Beam No.	Deflection f (in.)	Central load causing deflection N (lb.)	Ultimate central load N_w (lb.)	$\frac{N}{N_w}$	Neutral axis x (in.)	Stresses at failure		Ultimate bending moments		Central load causing cracks N_r (lb.)	$\frac{N_w}{N_r}$	Bending stress at cracking R_r (lb. per sq. in.)
						in concrete (lb. per sq. in.)	in steel (lb. per sq. in.)	lb.-in.	%			
1	0·31	1430	1740	0·82	3·60	1490	406,000	39,700		1430	1·21	3020
2	0·18	1320	1760	0·75	2·05	2180	334,000	40,200	100	1320	1·33	2460
3	0·39	1650	1780	0·92	2·95	1710	378,000	40,700		1320	1·35	2570
4	0·37	660	1130	0·58	4·02	940	283,000	26,100	65·0	550	2·06	1230
5	0·65	880	1040	0·85	0·59	3580	159,000	23,900	59·5	660	1·57	1290
6	0·28	660	1170	0·57	3·44	1010	262,000	26,800	66·8	660	1·77	1470
7	0·36	660	1150	0·58	2·45	1210	230,000	26,400	65·6	550	2·08	1130
10	0·28	770	1090	0·71	2·79	1050	221,000	25,100	62·5	440	2·47	1010
12	0·29	770	1130	0·69	3·08	1050	253,000	25,800	64·5	660	1·70	1470
8	0·16	1210	1520	0·77						1210	1·26	
9	0·28	770	940	0·82						660	1·43	

LOSS OF PRESTRESS DUE TO HIGH TEMPERATURE.—Two types of loss may occur. (1) The wires may slip at each end of the prestressed element; this was observed only with short elements with pre-tensioned steel during heating. Its absence in the longer members may have been because the ends of these members were not heated. (2) Losses may be caused by the decrease in the value of the modulus of elasticity.

In specimens 4 and 5 of test No. 1 the modulus of elasticity decreased to 57·7 per cent. of that for specimens 1, 2, and 3. Before the test the prestress in specimens 4 and 5 was 2000 lb. per square inch. After the test the stress was $2000 \times 0{\cdot}577 = 1155$ lb. per square inch. Similar losses occurred in the composite beams (*Table* V); visible cracks occurred in the control specimens at tensile bending stresses of 2460 lb., 2570 lb., and 3020 lb. per square inch, and

in the heated specimens at bending stresses between 1010 lb. and 1465 lb. per square inch. These stresses are calculated from

$$R_r = \frac{M_r}{b.h_2.z} \qquad \ldots \ldots \qquad (6)$$

in which M_r is the bending moment at which cracks appear and the other symbols are as given in *Fig.* 12. The position of the neutral axis was determined from extensometer readings.

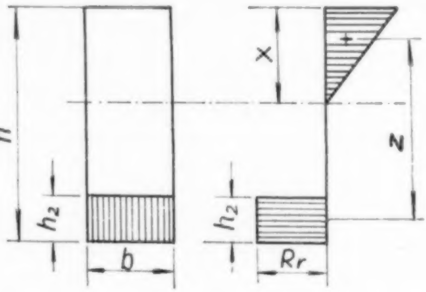


Fig. 12.

Increase of Strength and Elastic Modulus of Columns.

The strength of columns with pre-tensioned steel was greater after heating than that of unstressed columns (*Table* I). This is probably due to the use of wires; tests previously carried out in Poland indicate that these collaborate with the concrete to a greater extent than the bars used in reinforced concrete. The stresses in, and forces on, specimens 4 and 5 may be calculated from formulæ which follow.

$$R_w = \frac{N}{F_b(1 + n.\mu_z)} \qquad \ldots \ldots \qquad (7)$$

$$R_r = n.R_w \qquad \ldots \ldots \qquad (8)$$

and

$$N = F_b.R_w + \mu_z.F_b.R_r \qquad \ldots \ldots \qquad (9)$$

in which $n = \frac{E_{zt}}{E_{st}}$, E_{st} is the modulus of elasticity of the prestressed column, E_{zt} the modulus of elasticity of the steel under triaxial stress, R_w the stress in the concrete, R_r the stress in the steel, F_b the area of the section, μ_z the percentage of steel, and N the load. E_{zt} can be calculated from

$$E_{zt} = \frac{E_{st}(1 + \mu_z) - E_b}{\mu_z} \qquad \ldots \ldots \qquad (10)$$

when E_{st} and E_b are known. In the present case, from *Table* III, E_{st} for specimens 4 and 5 is 4,330,000 lb. per square inch, and E_b for specimen 11 is 2,040,000 lb. per square inch. F_b is 2·44 sq. in., μ_z is 0·0114, and hence

$$E_{zt} = \frac{4{,}330{,}000(1·0114) - 2{,}040{,}000}{0·0114} = 205{,}000{,}000 \text{ lb. per sq. in.}$$

and $$n = \frac{205,000,000}{4,330,000} = 47 \cdot 3.$$

If $R_w = 3650$ lb. per square inch, $N = 3650 \times 2 \cdot 44 \times (1 + 47 \cdot 3 \times 0 \cdot 0114) = 13,600$ lb. The actual ultimate load on specimens 4 and 5 was 14,750 lb., which exceeds the predicted value by 8·5 per cent.

Formulæ (7) and (8) may also be used to calculate R_w and R_r at failure for specimens 4 and 5; the values obtained are $R_w = 3910$ and $R_r = 185,000$ lb. per square inch. Formulæ (8) and (9) are based on the assumption that prestressed concrete behaves elastically until the ultimate strength is reached. In reality the stress–strain relationship is nearly linear. If it is assumed that the concrete in specimens 4 and 5 is as strong as in specimen 11, the concrete can resist a force $N_b = R_w . F_b = 3650 \times 2 \cdot 44 = 8900$ lb., and the remainder, N_z, which is 5850 lb., is resisted by the steel; the stress in the steel is 223,500 lb. per square inch. This is much less than the strength of the same steel before heating; this is because the steel undergoes structural changes at temperatures exceeding 350 deg. C., which reduce its strength. According to M. Guyon the strength of 0·1-in. hot rolled steel is reduced by 55 per cent. at 350 deg. C., and by 12 per cent. after cooling to the original temperature.

(1) A. W. Hill. "The Influence of Abnormal Temperatures on Prestressed Concrete." F.I.P. First International Congress, London, 1953.
(2) F. Leonhardt. "Spannbeton für die Praxis." Berlin. 1955.

Losses of Prestressing Force.

We are informed that the cables described as "Freyssinet cables" in the article on "Losses of Prestressing Force", by Dr. Jerzy Zielinski, in this journal for May last were not supplied by, or made under a licence issued by, Société Technique pour l'Utilisation de la Précontrainte, of Paris, which is the organisation appointed to manage and develop M. Freyssinet's patents. The cables and tensioning equipment were made and used in Poland without reference to S.T.U.P.

Lectures on Roads.

Courses of lectures on road materials and the construction of roads, including concrete roads, will be held at the Road Research Laboratory during the autumn and winter of 1959. Details may be had from the Director, Road Research Laboratory, Harmondsworth, Middlesex.

A Congress at Rotterdam.

A congress of the International Council for Building Research Studies and Documentation will be held in Rotterdam from September 21 to 26, 1959. The subjects to be discussed include housing, factors of safety, standardisation of dimensions, the use of large precast members, flat roofs, and insulation. The congress is open to all interested persons. Further details may be obtained from the Congress Secretariat, c/o Bouwcentrum, Weena 700, P.O. Box 299, Rotterdam, The Netherlands.

A Partnership.

Mr. Cyril Blumfield has taken Mr. Derek Lambert and Mr. Albert Witchlow into partnership. The consulting engineering practice will be carried on in the name of Cyril Blumfield and Partners at 82 Victoria Street, London, S.W.1, and 32 Corkran Road, Surbiton, Surrey.

Book Reviews.

"The Design of Prismatic Structures." By A. J. Ashdown. (London: Concrete Publications, Ltd. 1958. Price 9s.)

THIS book describes the analysis and design of roofs and similar structures comprising a series of planar slabs each of which is set at an angle to adjacent slabs. This form of construction is called "hipped-plate" in America and "faltewerke" in Germany. As in the previous edition, structures of one or more bays and structures continuous in a longitudinal direction are dealt with in this second edition, but minor revisions have been made to bring the work up to date. New matter includes the analysis of a prismatic-slab roof in which the angle between adjacent slabs is much less than 30 deg., that is the profile of the roof is nearly a curve. Examples are given of the design of roofs and other structures such as trough-shaped bunkers. A design for a prestressed concrete edge-beam for a prismatic roof is also given. Prismatic structures with sloping ends and with transverse stiffeners are dealt with as before.

No information is given regarding the erection of this form of structure, the economy of which is as much dependent on the method of construction, and consequent convenience of arranging the reinforcement, as on the refinements of design. A point emphasised by the author, and which is confirmed by a study of the examples, is that the relative cheapness of this form of structure is in part due to the smaller amount of design work (compared with "shells" and similar light long-span roofs) which results from using the procedures and formulæ given in this book.

"Berechnung von Flächengründungen." By Manfred Kanz. (Berlin: Wilhelm Ernst & Sohn. 1959. 35 D.M.)

A GOOD knowledge of the German language is necessary to understand this concise description of a method of designing solid raft foundations. Diagrams are given to provide a rapid means of calculating bending moments, settlement, and the pressure on the ground, and clear numerical examples are provided.

"The Design of Shells." By A. Chronowicz. (London: Crosby Lockwood & Son, Ltd. 1959. Price 42s.)

THE sub-title of this book, "A Practical Approach", is fully justified because it is based on work actually carried out in a commercial drawing office—it is rarely that a reader is given an insight into the methods used in such offices for complex structures.

The book deals only with cylindrical "shell" roofs, and the theory and mathematical treatment are dealt with in an admirably simple manner. The "membrane" forces are derived in the usual way and Finsterwalder's method is used to reconcile the strains at the edges of the vault, but the labour involved in the rigorous development of the subsequent equations is lessened by incorporating a degree of tolerance based on the exact roots of the characteristic equation instead of some of the approximations suggested by other writers. The author makes the higher mathematics easier to understand by explaining the formulation of the boundary equations and the evaluation by numerical treatment of the constants of integration. Other problems, such as north-light roofs, prestressed edge-beams, roofs with transverse ribs, and end-frames are dealt with in a similar manner. The similarity of polygonal roofs and shell roofs, and the application of the column-analogy method to the design of shell roofs, are among several other aspects discussed. The author introduces a "balanced-shear" method for the preliminary design (and for checking designs or even for designing simple vaults), and gives valid reasons for preferring this method to the ordinary beam method or other methods.

An extensive bibliography is given, but the author's further comments, such as those in the preface, on selected texts are more valuable. The diagrams are exceptionally well drawn and most of the photographs are of structures with the design of which the author has been personally concerned; throughout the book it is clear that he writes only what he knows, and his experience relates to most types of shell roofs commonly used.

A Sixteen-story Office Building in London.

A FEATURE of an office building sixteen stories high now being erected in High Holborn, London, is that the outer columns are 10 in. outside the faces of the two main parts of the building (*Fig.* 1). The floors bear on upstand beams which are continuous around the building and are attached to concrete brackets on the columns. All other beams are concealed in the hollow-tile floors which are 10½ in. thick. The lift-shafts and service-ducts at each end of the central part of the building have reinforced concrete walls which stiffen the building vertically. Two parts eight stories high are built with the columns inside the walls, and an extension at the front of the building has two stories which are connected to the basement by a circular stair.

The edge-beams supporting the floors in the main blocks are 3 ft. 6 in. deep and have external facings 4 in. thick of precast concrete slabs which were made with Portland stone aggregate and were used as permanent shuttering. The procedure for forming the beams was as follows. The floor was cast with the beam reinforcement in place. When it had hardened a supporting frame for the precast slabs was erected on the scaffolding (*Fig.* 2) and the internal shuttering was erected on the floor. The slabs were placed in the frame and ties were placed in the mortar joints between them to fix them to the beam. The concrete was then placed so that the beam, the bracket, and part of the supporting column were cast monolithically. The bracket was designed to resist shearing forces only, no bending moments being transmitted to the columns except those produced by eccentric loads. The concrete for the whole structure above ground was a 1 : 1½ : 3 mixture with ¾-in. graded aggregate; the maximum slump was 2 in. for the concrete in the floor and beams and 1½ in. for the concrete in the columns.

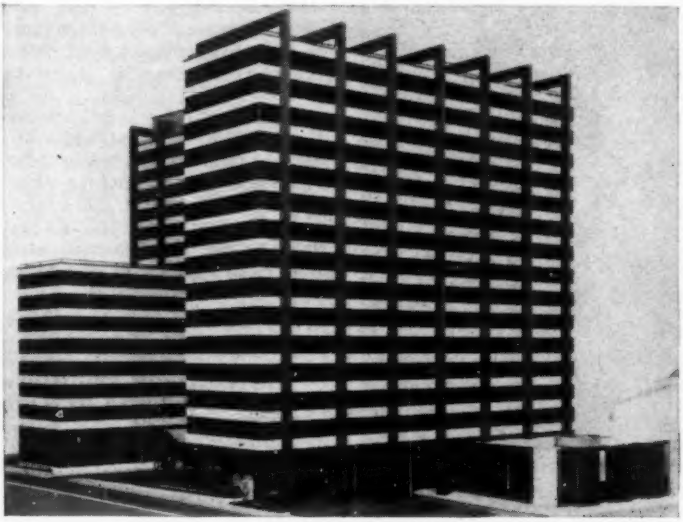


Fig. 1.

One of the reinforced concrete stairs was built with the steps and landings cantilevered from a central wall and is therefore structurally independent. Reinforcement is provided to act as ties between the landings and the floors. At times during construction the stairs extended above the rest of the completed work and were used to give access to the floor on which work was in progress.

To obtain the smooth soffit specified

for a circular staircase in the two-story building, the wooden shuttering was covered with a smooth layer of plaster the surface of which was treated with oil.

The roof of the basement is designed to support 2 ft. of earth for a garden and a road which passes under the main building; a 1 : 2 : 4 mixture with ¾-in. graded aggregate and a slump of ½ in. was used and compacted by immersion vibrators. A plastic water-bar was set in the construction joints, which occur at intervals of 10 ft. or 12 ft. and a rebate was formed so that any cracking of the concrete due to shrinking would take place at these joints.

Fig. 2.

The architects are Messrs. Trehearne and Norman Preston and Partners, the consulting engineers are Messrs. R. Travers Morgan & Partners, and the general contractors are Tersons, Ltd.

Metric Conversion Factors.

British Standard No. 350, "Conversion Factors and Tables", which has previously been issued as one volume, is now being issued in two parts. The first part, which was recently published by the British Standard Institution at a price of 15*s*., describes the British and metric units of measurement used in connection with metrology, mechanics, heat, and associated branches of physics. Fifty tables are given relating similar units one with another, but reference tables from which the number of British units of one kind can be readily converted to the equivalent number of metric units (or vice versa) are to be given in Part 2. The minute differences between the U.S.A. and British units of length and weight are given, as well as the more significant differences in volumetric measures.

The Effects of Admixtures on Concrete Placed by Tremie.

Laboratory tests on concrete placed under water by tremie are described by Mr. J. Wayman Williams, Jr., in the Journal of the American Concrete Institute for February, 1959. It is stated that the addition to the concrete of half a fluid ounce of a neutralised Vinsol-resin air-entraining compound and 4 fluid ounces of a retarder with an adipic acid base to each 94 lb. of cement reduced the rate of generation of heat, retarded initial setting, and reduced the maximum temperature. Laitance was reduced by about 80 per cent., and the variation of strength of the concrete by about 75 per cent. The flow and cohesiveness of the concrete were improved.

Bending Moments on Continuous Beams Calculated by Nomogram.

By J. W. WHITE, B.Sc., A.M.I.C.E., A.M.I.Struct.E.

THE nomogram on pages 246 and 247 is for determining the degree of fixity f° and rigidity R_n when calculating the bending moments on continuous beams by the degree-of-fixity method. The expressions for these factors are

$$R_n = \frac{K_n}{1 - 0{\cdot}25 f^\circ_n} \quad \text{and} \quad f^\circ_{+1} = \frac{R_n}{R_n + K_{n+1}}$$

which are the components of the general formula

$$f^\circ_{n+1} = \frac{1}{1 + \frac{K_{n+1}}{K_n}\left(1 - \frac{f^\circ_n}{4}\right)}.$$

This is formula (5) in "Continuous Beam Structures"* by E. Shepley, in which the derivation of the formula is given.

EXAMPLE NO. 1.—*Fig.* 1 shows the solution of a problem of a beam continuous over three unequal spans, one end being fixed and the other freely supported. The operation of the moment-distribution process and the use of the nomogram in two stages are self-explanatory.

EXAMPLE NO. 2.—*Fig.* 2 gives the solution for a beam continuous over two unequal spans and cantilevering beyond the support at one end and partially

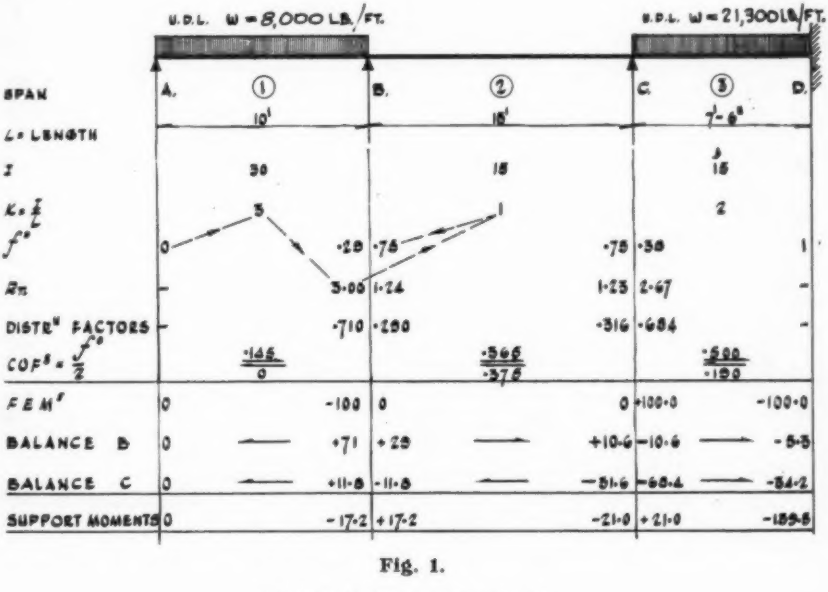
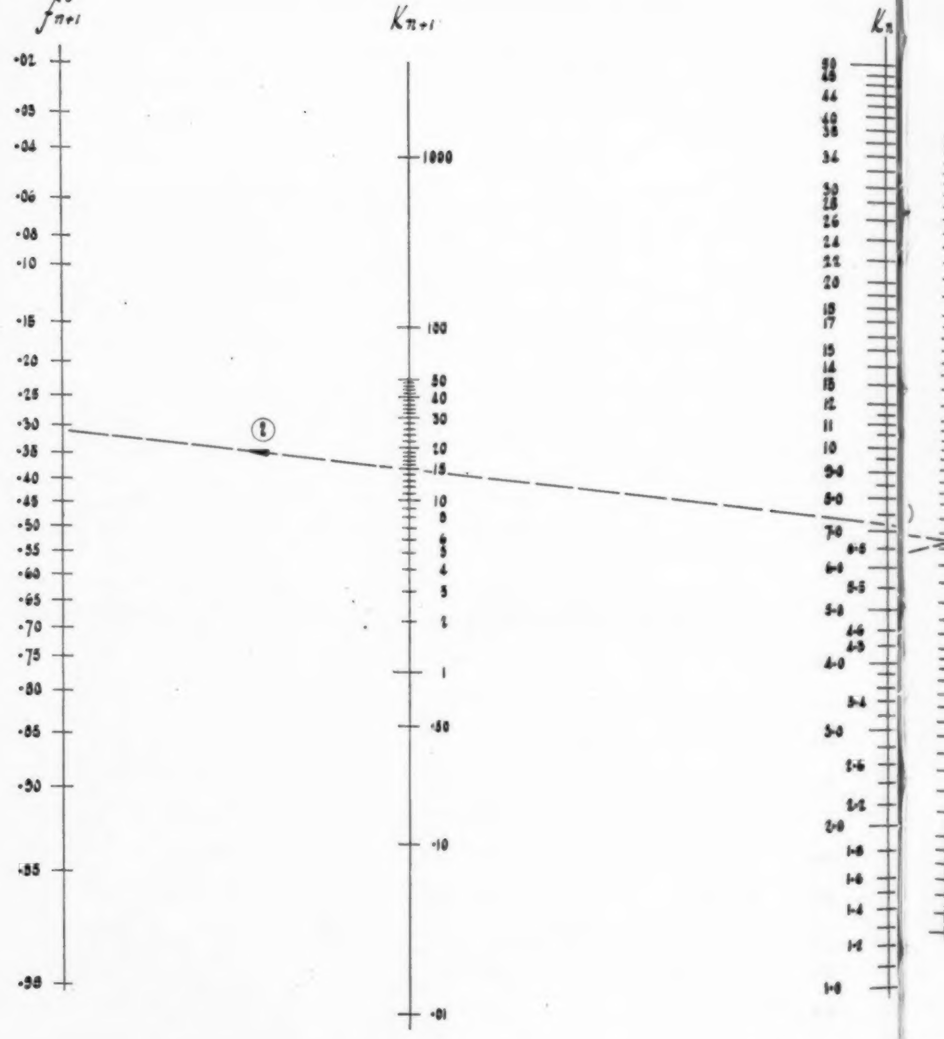
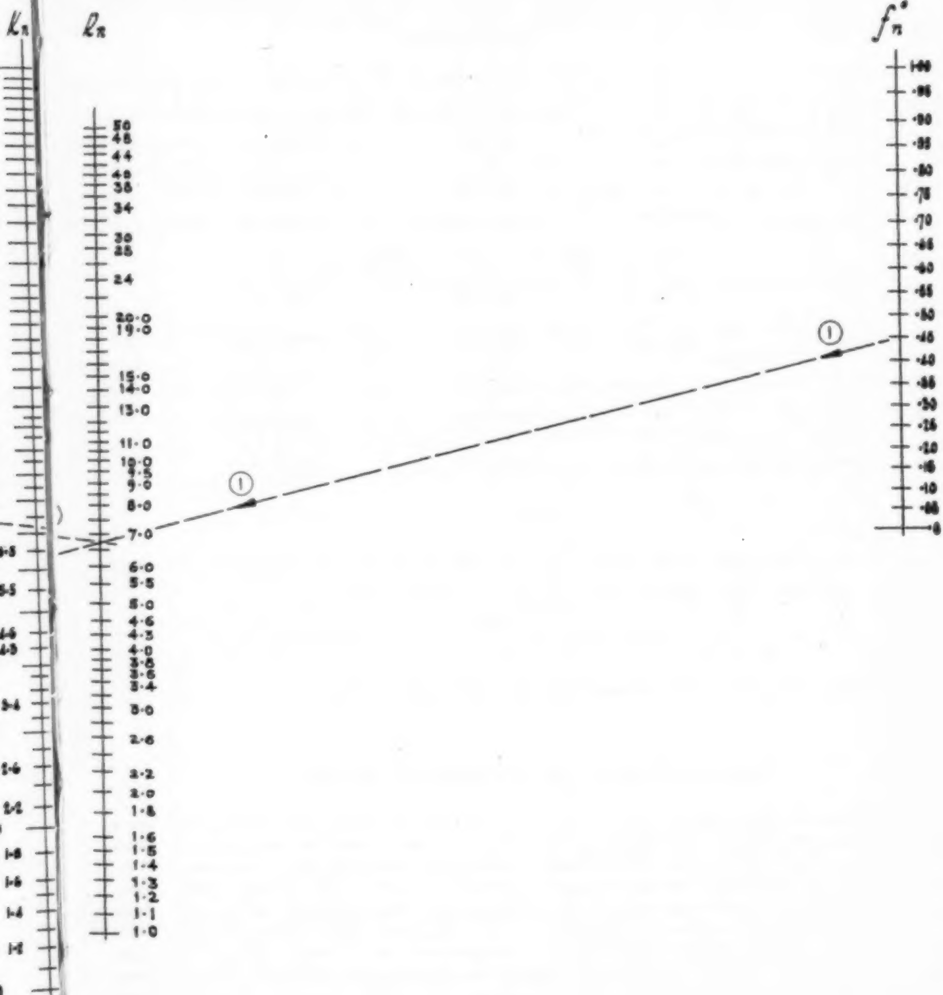


Fig. 1.

* Published by Concrete Publications Ltd.



Nomogram for Bending ‌**ts c**



nding [illegible]ts on Continuous Beams.

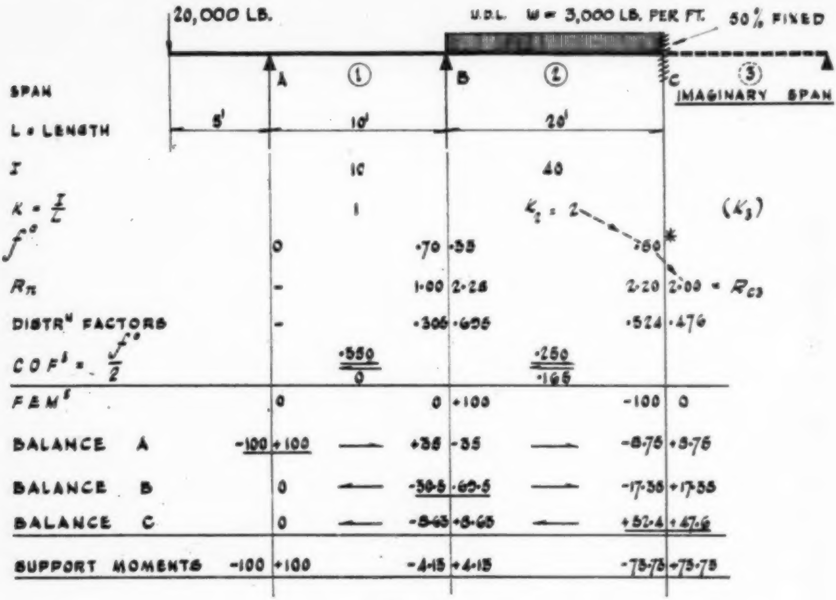


Fig. 2.

fixed at the other end. The degree-of-fixity factor of 0·5 at support C (marked with an asterisk) is assumed, and R_{c3} is calculated from

$$f^{\circ}_{c2} = \frac{R_{c3}}{R_{c3} + K_2}, \text{ that is } 0{\cdot}5 = \frac{R_{c3}}{R_{c3} + 2}, \text{ from which } R_{c3} = 2.$$

In other respects the calculations are self-explanatory.

Effect of Fatigue on Prestressed Concrete.

Examination of the behaviour of prestressed concrete under repeated loading has now reached the stage where conclusions may be drawn for beams stressed with pre-tensioned hard-drawn steel wire or with post-tensioned cables of this wire and grouted. An important conclusion is that even several million repetitions of loading within the normal working range have no significant effect on the later performance of the beams.

When fatigue failure occurs by fracture of pre-tensioned wires, the ratio of the maximum load causing failure after one million repetitions to the loads causing failure under static conditions has been found to vary from 0·70 to 0·85 for plain wires and from 0·65 to 0·75 for crimped or indented wires. In beams with post-tensioned and grouted cables of plain wire, this ratio had values of about 0·8 irrespective of whether failure was the result of fatigue of the steel or of the concrete. Since the working load of beams is not usually greater than one-half of the ultimate static strength, the chance of failure due to fatigue is small. [From "Building Research, 1957." Published by H.M. Stationery Office. Price 5*s.* 6*d.*]

Design of Helical Staircases—2.*

Statically-Indeterminate Cases.

By JACQUES S. COHEN.

Applications.

CIRCULAR HELICAL BEAM WITH BOTH ENDS FIXED (*Fig.* 11).

A member in the form of a circular helical arc fixed at both ends is six times indeterminate; there are six equations of equilibrium and twelve unknown reactions. The deformation equations are used to determine the unknown reactions. The conditions of restraint at the two supports are such that there is no displacement or angular rotation at A and B. Therefore

$$\left.\begin{aligned}\psi_{t_A} = \psi_{n_A} = \psi_{b_A} = D_{t_A} = D_{n_A} = D_{b_A} = 0\\ \psi_{t_B} = \psi_{n_B} = \psi_{b_B} = D_{t_B} = D_{n_B} = D_{b_B} = 0\end{aligned}\right\} . \quad . \quad . \quad (48)$$

From equations (45) and (47) and conditions (48), twelve simultaneous equations having as " unknowns " the twelve constants of differentiation C_1 to C_{12} are obtained. The solution of these gives the twelve constants, which can therefore be used in equations (24) to obtain the internal forces and moments at any point, or in equations (45) and (47) to obtain the deformations at any point.

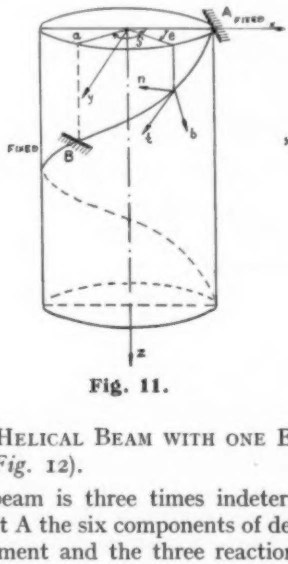


Fig. 11.

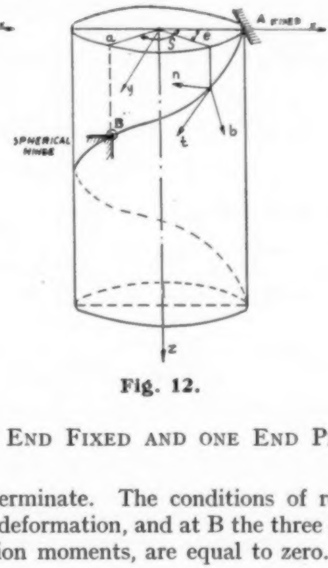


Fig. 12.

CIRCULAR HELICAL BEAM WITH ONE END FIXED AND ONE END PIN JOINTED (*Fig.* 12).

This beam is three times indeterminate. The conditions of restraint are such that at A the six components of deformation, and at B the three components of displacement and the three reaction moments, are equal to zero. Therefore

$$\left.\begin{aligned}&\psi_{t_A} = \psi_{n_A} = \psi_{b_A} = D_{t_A} = D_{n_A} = D_{b_A} = 0.\\ &D_{t_B} = D_{n_B} = D_{b_B} = 0.\\ &M_{t_B} = M_{n_B} = M_{b_B} = 0.\end{aligned}\right\} \quad . \quad . \quad (49)$$

* *Continued from June, 1959.*

From equations (45), (47), and (24), and conditions (49), twelve simultaneous equations having as unknowns the twelve constants of differentiation C_1 to C_{12} are obtained. The solution of these gives the twelve constants; the internal forces and moments at any point can then be found from (24) and the deformations from (45) and (47).

To illustrate these two cases, the staircase in *Fig.* 6 with both ends simply supported will be re-designed with restrained ends; a comparison of the designs will then show the effect of restraint at the supports on the internal forces and moments at any point.

Circular Helical Beam Fixed at Both Ends.

Inserting $\theta = 0$ and $\theta = \delta$ in equations (45) and (47), and inserting these in conditions (48), leads to equations (50) (see p. 251).

Values of C_7 to C_{12} are obtained respectively from equations [50(a), (c), (e), (g), (i), and (k)], and inserted in the remaining equations to obtain equations (51).

In equations (51) there are only six "unknowns", as A_1 to A_6 and B_1 to B_6 are obtained from (44) and (46), and the values of M_{t_A}, M_{t_B}, M_{n_A}, M_{n_B} for $\theta = 0$ and $\theta = \delta$ are as follows [from equations (24)]:

$$M_{t_A} = C_4 + C_6;$$

$$M_{t_B} = C_4 + C_5 \sin\delta + C_6 \cos\delta + a\delta\cot\phi(C_2\cos\delta - C_3\sin\delta) + wa^2\delta;$$

$$M_{n_A} = \frac{1}{\sin\phi}(C_5 + a\cot\phi\, C_2 + wa^2 + ma);$$

$$M_{n_B} = \frac{1}{\sin\phi}[C_5\cos\delta - C_6\sin\delta + a\cot\phi\{C_2(\cos\delta - \delta\sin\delta) - C_3(\sin\delta + \delta\cos\delta)\} + wa^2 + ma].$$

Substituting these values in equations (51), multiplying the first three equations by $\dfrac{\sin^2\phi}{K_1 a^2\cos\phi}$ and the last three by $\dfrac{\sin^2\phi}{K_1 a^2\cos\phi} \times \dfrac{\sin\phi\cos\phi}{a}$, and rearranging, equations (52) with "unknowns" C_2 to C_6, are obtained. In these equations

$$\left.\begin{array}{l} \varepsilon = \dfrac{K_2}{K_1},\quad \varepsilon_1 = \sigma\cos^2\phi + \varepsilon\sin^2\phi. \quad \varepsilon_2 = 2(1+\sigma) + \varepsilon. \\ \quad \varepsilon_3 = 1 + \sigma\sin^2\phi + \varepsilon\cos^2\phi. \quad \varepsilon_4 = (\sigma - \varepsilon)\sin\phi\cos\phi = \dfrac{\sin 2\phi}{2}(\sigma - \varepsilon). \end{array}\right\}. \qquad (53)$$

In the special case of symmetrical loading, the solution may be simplified considerably. This is represented by using equations (31) for $\theta = 0$ and $\theta = \delta$ at the two supports; if equations (24) and (31) are combined, the six equations (54) (see p. 255) are obtained. From (54) four of the six constants are found:

$$\left.\begin{array}{l} C_1 = -\dfrac{wa}{2}\delta\cot\phi. \qquad C_2 = \dfrac{-C_5\sin\delta - C_6(1+\cos\delta)}{a\delta\cot\phi} = -C_3\cot\dfrac{\delta}{2}. \\ C_3 = \dfrac{+C_5(1-\cos\delta) + C_6\sin\delta}{a\delta\cot\phi} = -C_2\tan\dfrac{\delta}{2}. \qquad C_4 = -\dfrac{wa^2\delta}{2}. \end{array}\right\}. \qquad (55)$$

$$
\left.
\begin{aligned}
&\text{(a)}\quad \psi_{t_a} = 0 = C_7 + C_9.\\
&\text{(b)}\quad \psi_{t_b} = 0 = C_7 + C_8 \sin\delta + C_9\cos\delta + A_1\delta - A_2\frac{\delta\sin\delta}{2} - A_3\frac{\delta\cos\delta}{2}\\
&\qquad + A_4\left(\frac{-\delta^2\sin\delta - 3\delta\cos\delta}{4}\right) + A_5\left(\frac{-\delta^2\cos\delta + 3\delta\sin\delta}{4}\right) + A_6\frac{\delta^2}{2}.\\
&\text{(c)}\quad \psi_{n_a} = 0 = C_8 + A_1 - \frac{A_3}{2} - \frac{3}{4}A_4 - \frac{K_1\sigma a}{\sin\phi}M_{t_a}.\\
&\text{(d)}\quad \psi_{n_b} = 0 = C_8\cos\delta - C_9\sin\delta + A_1 - A_2\left(\frac{\delta\cos\delta + \sin\delta}{2}\right)\\
&\qquad - A_3\left(\frac{-\delta\sin\delta + \cos\delta}{2}\right) + A_4\left(\frac{-\delta^2\cos\delta + \delta\sin\delta - 3\cos\delta}{4}\right)\\
&\qquad + A_5\left(\frac{\delta^2\sin\delta + \delta\cos\delta + 3\sin\delta}{4}\right) + A_6\delta - \frac{K_1\sigma a}{\sin\phi}M_{t_b}.\\
&\text{(e)}\quad \psi_{b_a} = 0 = -C_9 - A_2 + A_5 + A_6 - K_1 a(1+\sigma)M_{n_a}.\\
&\text{(f)}\quad \psi_{b_b} = 0 = -C_8\sin\delta - C_9\cos\delta - A_2\left(\frac{2\cos\delta - \delta\sin\delta}{2}\right)\\
&\qquad - A_3\left(\frac{-2\sin\delta - \delta\cos\delta}{2}\right) + A_4\left(\frac{\delta^2\sin\delta - \delta\cos\delta + 4\sin\delta}{4}\right)\\
&\qquad + A_5\left(\frac{\delta^2\cos\delta + \delta\sin\delta + 4\cos\delta}{4}\right) + A_6 - K_1 a(1+\sigma)M_{n_b}.\\
&\text{(g)}\quad D_{t_a} = 0 = C_{10} + C_{12}.\\
&\text{(h)}\quad D_{t_b} = 0 = C_{10} + C_{11}\sin\delta + C_{12}\cos\delta + B_1\delta - B_2\frac{\delta\sin\delta}{2} - B_3\frac{\delta\cos\delta}{2}\\
&\qquad + B_4\left(\frac{-\delta^2\sin\delta - 3\delta\cos\delta}{4}\right) + B_5\left(\frac{-\delta^2\cos\delta + 3\delta\sin\delta}{4}\right)\\
&\qquad + B_6\frac{\delta^2}{2} + B_7\left(\frac{-\delta^3\cos\delta}{6} + \frac{3\delta^2\sin\delta}{4} + \frac{7\delta\cos\delta}{4}\right)\\
&\qquad + B_8\left(\frac{-\delta^3\sin\delta}{6} - \frac{3\delta^2\cos\delta}{4} + \frac{7\delta\sin\delta}{4}\right).\\
&\text{(i)}\quad D_{n_a} = 0 = C_{11} + B_1 - \frac{B_3}{2} - \frac{3}{4}B_4 + \frac{7}{4}B_7.\\
&\text{(j)}\quad D_{n_b} = 0 = C_{11}\cos\delta - C_{12}\sin\delta + B_1 - B_2\left(\frac{\delta\cos\delta + \sin\delta}{2}\right)\\
&\qquad - B_3\left(\frac{-\delta\sin\delta + \cos\delta}{2}\right) + B_4\left(\frac{-\delta^2\cos\delta + \delta\sin\delta - 3\cos\delta}{4}\right)\\
&\qquad + B_5\left(\frac{\delta^2\sin\delta + \delta\cos\delta + 3\sin\delta}{4}\right) + B_6\delta\\
&\qquad + B_7\left(\frac{\delta^3\sin\delta}{6} + \frac{\delta^2\cos\delta}{4} - \frac{\delta\sin\delta}{4} + \frac{7}{4}\cos\delta\right)\\
&\qquad + B_8\left(-\frac{\delta^3\cos\delta}{6} + \frac{\delta^2\sin\delta}{4} + \frac{\delta\cos\delta}{4} + \frac{7}{4}\sin\delta\right).\\
&\text{(k)}\quad D_{b_a} = 0 = -C_{12} - B_2 + B_5 + B_6 + 2B_8.\\
&\text{(l)}\quad D_{b_b} = 0 = -C_{11}\sin\delta - C_{12}\cos\delta - B_2\left(\frac{-\delta\sin\delta + 2\cos\delta}{2}\right)\\
&\qquad - B_3\left(\frac{-\delta\cos\delta - 2\sin\delta}{2}\right) + B_4\left(\frac{\delta^2\sin\delta - \delta\cos\delta + 4\sin\delta}{4}\right)\\
&\qquad + B_5\left(\frac{\delta^2\cos\delta + \delta\sin\delta + 4\cos\delta}{4}\right) + B_6\\
&\qquad + B_7\left(\frac{\delta^3\cos\delta}{6} + \frac{\delta^2\sin\delta}{4} + \frac{\delta\cos\delta}{4} - 2\sin\delta\right)\\
&\qquad + B_8\left(\frac{\delta^3\sin\delta}{6} - \frac{\delta^2\cos\delta}{4} + \frac{\delta\sin\delta}{4} + 2\cos\delta\right).
\end{aligned}
\right\}
\tag{50}
$$

$$
\left.
\begin{aligned}
&\text{(a)}\ \left(-A_1+\frac{A_3}{2}+\frac{3}{4}A_4+\frac{K_1\sigma a}{\sin\phi}M_{t_A}\right)\frac{\sin\delta}{\delta}\\
&\quad+(-A_2+A_5+A_6-K_1a(1+\sigma)M_{n_A})\,\frac{\cos\delta-1}{\delta}+A_1\\
&\quad-A_2\frac{\sin\delta}{2}-A_3\frac{\cos\delta}{2}+A_4\left(\frac{-\delta\sin\delta-3\cos\delta}{4}\right)\\
&\quad+A_5\left(\frac{-\delta\cos\delta+3\sin\delta}{4}\right)+A_6\frac{\delta}{2}=0.\\
&\text{(b)}\ \left(-A_1+\frac{A_3}{2}+\frac{3}{4}A_4+\frac{K_1\sigma a}{\sin\phi}M_{t_A}\right)\cos\delta\\
&\quad+[A_2-A_5-A_6+K_1a(1+\sigma)M_{n_A}]\sin\delta+A_1\\
&\quad-A_2\left(\frac{\delta\cos\delta+\sin\delta}{2}\right)-A_3\left(\frac{-\delta\sin\delta+\cos\delta}{2}\right)\\
&\quad+A_4\left(\frac{-\delta^2\cos\delta+\delta\sin\delta-3\cos\delta}{4}\right)\\
&\quad+A_5\left(\frac{\delta^2\sin\delta+\delta\cos\delta+3\sin\delta}{4}\right)+A_6\delta-\frac{K_1\sigma a}{\sin\phi}M_{t_B}=0.\\
&\text{(c)}\ \left(A_1-\frac{A_3}{2}-\frac{3}{4}A_4-\frac{K_1\sigma a}{\sin\phi}M_{t_A}\right)\sin\delta\\
&\quad+[A_2-A_5-A_6+K_1a(1+\sigma)M_{n_A}]\cos\delta\\
&\quad-A_2\left(\frac{2\cos\delta-\delta\sin\delta}{2}\right)-A_3\left(\frac{-2\sin\delta-\delta\cos\delta}{2}\right)\\
&\quad+A_4\left(\frac{\delta^2\sin\delta-\delta\cos\delta+4\sin\delta}{4}\right)\\
&\quad+A_5\left(\frac{\delta^2\cos\delta+\delta\sin\delta+4\cos\delta}{4}\right)+A_6-K_1a(1+\sigma)M_{n_B}=0.\\
&\text{(d)}\ \left(-B_1+\frac{B_3}{2}+\frac{3}{4}B_4-\frac{7}{4}B_7\right)\frac{\sin\delta}{\delta}+(-B_2+B_5+B_6+2B_8)\,\frac{\cos\delta-1}{\delta}\\
&\quad+B_1-B_2\frac{\sin\delta}{2}-B_3\frac{\cos\delta}{2}+B_4\left(\frac{-\delta\sin\delta-3\cos\delta}{4}\right)\\
&\quad+B_5\left(\frac{-\delta\cos\delta+3\sin\delta}{4}\right)+B_6\frac{\delta}{2}+B_7\left(\frac{-\delta^2\cos\delta}{6}+\frac{3\delta\sin\delta}{4}+\frac{7\cos\delta}{4}\right)\\
&\quad+B_8\left(-\frac{\delta^2\sin\delta}{6}-\frac{3\delta\cos\delta}{4}+\frac{7\sin\delta}{4}\right)=0.\\
&\text{(e)}\ \left(-B_1+\frac{B_3}{2}+\frac{3}{4}B_4-\frac{7}{4}B_7\right)\cos\delta+(+B_2-B_5-B_6-2B_8)\sin\delta\\
&\quad+B_1-B_2\left(\frac{\delta\cos\delta+\sin\delta}{2}\right)-B_3\left(\frac{-\delta\sin\delta+\cos\delta}{2}\right)\\
&\quad+B_4\left(\frac{-\delta^2\cos\delta+\delta\sin\delta-3\cos\delta}{4}\right)\\
&\quad+B_5\left(\frac{\delta^2\sin\delta+\delta\cos\delta+3\sin\delta}{4}\right)+B_6\delta\\
&\quad+B_7\left(\frac{\delta^3\sin\delta}{6}+\frac{\delta^2\cos\delta}{4}-\frac{\delta\sin\delta}{4}+\frac{7}{4}\cos\delta\right)\\
&\quad+B_8\left(\frac{-\delta^3\cos\delta}{6}+\frac{\delta^2\sin\delta}{4}+\frac{\delta\cos\delta}{4}+\frac{7}{4}\sin\delta\right)=0.\\
&\text{(f)}\ \left(B_1-\frac{B_3}{2}-\frac{3}{4}B_4+\frac{7}{4}B_7\right)\sin\delta+(B_2-B_5-B_6-2B_8)\cos\delta\\
&\quad-B_2\left(\frac{-\delta\sin\delta+2\cos\delta}{2}\right)-B_3\left(\frac{-\delta\cos\delta-2\sin\delta}{2}\right)\\
&\quad+B_4\left(\frac{\delta^2\sin\delta-\delta\cos\delta+4\sin\delta}{4}\right)+B_5\left(\frac{\delta^2\cos\delta+\delta\sin\delta+4\cos\delta}{4}\right)\\
&\quad+B_6+B_7\left(\frac{\delta^3\cos\delta}{6}+\frac{\delta^2\sin\delta}{4}+\frac{\delta\cos\delta}{4}-2\sin\delta\right)\\
&\quad+B_8\left(\frac{\delta^3\sin\delta}{6}-\frac{\delta^2\cos\delta}{4}+\frac{\delta\sin\delta}{4}+2\cos\delta\right)=0.
\end{aligned}
\right\}\qquad(51)
$$

$$
\begin{aligned}
\text{(a)}\quad & \varepsilon \tan^2\phi\left(\frac{\sin\delta}{\delta}-1\right)C_1+\left[\varepsilon_1\frac{\cos\delta-1}{\delta}+\sin\delta\left(\frac{\varepsilon_2}{2}-\frac{3}{4}\varepsilon_3\right)+\varepsilon_3\frac{\delta\cos\delta}{4}\right]C_2 \\
& +\left[\left(-\frac{\varepsilon_2}{2}+\frac{3}{4}\varepsilon_3\right)\frac{\sin\delta}{\delta}+\cos\delta\left(\frac{\varepsilon_2}{2}-\frac{3}{4}\varepsilon_3\right)-\frac{\varepsilon_3}{4}\delta\sin\delta\right]C_3 \\
& +\frac{\tan\phi}{a}\left[\frac{\sin\delta}{\delta}(\sigma-\varepsilon)\sin^2\phi+\varepsilon_1\right]C_4+\frac{\tan\phi}{a}\Bigg[\frac{\cos\delta-1}{\delta}(\varepsilon-\sigma)\cos^2\phi \\
& +\varepsilon_3\frac{\sin\delta}{2}\Bigg]C_5+\frac{\tan\phi}{a}\left[\left(-\frac{\varepsilon_3}{2}+\sigma\right)\frac{\sin\delta}{\delta}+\frac{\varepsilon_3}{2}\cos\delta\right]C_6 \\
& +wa\left(\varepsilon_4\frac{\delta}{2}-\varepsilon_3\frac{\cos\delta-1}{\delta}\tan\phi\right)-m(1+\sigma)\frac{\cos\delta-1}{\delta}\tan\phi=0.
\end{aligned}
$$

$$
\begin{aligned}
\text{(b)}\quad & \varepsilon\tan^2\phi(\cos\delta-1)C_1+\Bigg[\left(-\frac{\varepsilon}{2}+\frac{\varepsilon_3}{4}\right)\sin\delta+\left(1+\frac{\varepsilon}{2}-\frac{\varepsilon_3}{4}\right)\delta\cos\delta \\
& -\frac{\varepsilon_3}{4}\delta^2\sin\delta\Bigg]C_2+\left[\left(-1-\frac{\varepsilon}{2}+\frac{\varepsilon_3}{4}\right)\delta\sin\delta-\frac{\varepsilon_3}{4}\delta^2\cos\delta\right]C_3 \\
& +(\varepsilon-\sigma)\frac{\tan\phi}{a}(1-\cos\delta)\sin^2\phi\, C_4+\frac{\tan\phi}{a}\Bigg[\varepsilon_3\frac{\delta\cos\delta-\sin\delta}{2} \\
& +\sin\delta\Bigg]C_5-\frac{\tan\phi}{a}\varepsilon_3\frac{\delta\sin\delta}{2}C_6+wa\tan\phi[\varepsilon_3(\sin\delta-\delta)+\delta] \\
& +m(1+\sigma)\tan\phi\sin\delta=0.
\end{aligned}
$$

$$
\begin{aligned}
\text{(c)}\quad & -\varepsilon\tan^2\phi\sin\delta\, C_1+\left[-\delta\sin\delta.\left(\frac{\varepsilon}{2}+\frac{\varepsilon_3}{4}\right)-\frac{\varepsilon_3}{4}\delta^2\cos\delta\right]C_2 \\
& +\left[\left(-\frac{\varepsilon}{2}+\frac{\varepsilon_3}{4}\right)\sin\delta+\left(-\frac{\varepsilon}{2}-\frac{\varepsilon_3}{4}\right)\delta\cos\delta+\frac{\varepsilon_3}{4}\delta^2\sin\delta\right]C_3 \\
& +\frac{\tan\phi}{a}(\varepsilon-\sigma)\sin\delta\sin^2\phi\, C_4-\frac{\varepsilon_3}{2}\frac{\tan\phi}{a}\delta\sin\delta C_5 \\
& +\frac{\tan\phi}{a}\left[-\varepsilon_3\frac{\delta\cos\delta+\sin\delta}{2}+\sin\delta\right]C_6 \\
& -\tan\phi(1-\cos\delta)[wa\varepsilon_3+m(1+\sigma)]=0.
\end{aligned}
$$

$$
\begin{aligned}
\text{(d)}\quad & -\varepsilon\sin^2\phi\left[\frac{\sin\delta}{\delta}(2-\tan^2\phi)-\cos\delta+\tan^2\phi-1\right]C_1 \\
& +\Bigg[\frac{\cos\delta-1}{\delta}(\sigma\cos^2\phi-\varepsilon_1\cos 2\phi) \\
& +\frac{\sin\delta}{4}(3\sigma\cos^2\phi-\varepsilon\sin^2\phi-3\varepsilon_1\cos 2\phi)+(\varepsilon-\sigma)\,\delta\cos\delta\frac{\sin^2 2\phi}{8} \\
& -\delta^2\sin\delta\frac{\cos^2\phi}{12}\varepsilon_3\Bigg]C_2 \\
& +\Bigg[\left\{\frac{\cos^2\phi(5\sigma\sin^2\phi-1)+\varepsilon(\cos^2 2\phi+\sin^4\phi)}{4}\right\}\left(\cos\delta-\frac{\sin\delta}{\delta}\right) \\
& +\varepsilon_4\delta\sin\delta\frac{\sin 2\phi}{4}-\delta^2\cos\delta\frac{\cos^2\phi}{12}\varepsilon_3\Bigg]C_3 \\
& +\frac{\sin^2\phi}{a}\left[-\frac{\sin\delta}{\delta}(3\varepsilon_4+\varepsilon\tan\phi)+\varepsilon_4\cos\delta+2\varepsilon_4+\varepsilon\tan\phi\right]C_4
\end{aligned}
$$

Part of Equations (52).

$$
\begin{aligned}
&+\frac{\sin^2\phi}{a}\Big[-(2\varepsilon_4+\varepsilon\cot\phi)\frac{\cos\delta-1}{\delta}-\{5\varepsilon_4+(1+3\varepsilon)\cot\phi\}\frac{\sin\delta}{4}\\
&+\frac{\delta\cos\delta}{4}\varepsilon_3\cot\phi\Big]C_5+\frac{\sin^2\phi}{4a}\Big[\Big(\frac{\sin\delta}{\delta}-\cos\delta\Big)(3\varepsilon_3-2\varepsilon)\cot\phi\\
&-\varepsilon_3\delta\sin\delta\cot\phi\Big]C_6+wa\,\frac{\sin 2\phi}{2}\Big[\frac{\cos\delta-1}{\delta}(4\varepsilon_3-\varepsilon-2)\\
&+\varepsilon_3\sin\delta+\frac{\delta}{2}(2\sigma\sin^2\phi+\varepsilon\cos 2\phi)\Big]\\
&+m(1+\sigma)\frac{\sin 2\phi}{2}\Big(2\frac{\cos\delta-1}{\delta}+\sin\delta\Big)=0.
\end{aligned}
$$

(e)
$$
\begin{aligned}
&-\varepsilon\sin^2\phi[(1-\tan^2\phi)(\cos\delta-1)+\delta\sin\delta]C_1+\Big[-\frac{\sin\delta}{2}\varepsilon_1\sin^2\phi\\
&+\frac{\delta\cos\delta}{2}\varepsilon_1\sin^2\phi+\delta^2\sin\delta\frac{\cos^2\phi}{4}(\varepsilon_3-2\varepsilon-2)-\frac{\varepsilon_3}{12}\delta^3\cos\delta\cos^2\phi\Big]C_2\\
&+\Big[\frac{\delta\sin\delta}{4}(\cos^2\phi-\varepsilon-\varepsilon_1\sin^2\phi)+\frac{\delta^2\cos\delta}{4}(-\cos^2\phi-\varepsilon+\varepsilon_1\sin^2\phi)\\
&+\frac{\varepsilon_3}{12}\delta^3\sin\delta\cos^2\phi\Big]C_3-[(\cos\delta-1)(2\varepsilon_4+\varepsilon\tan\phi)+\varepsilon_4\delta\sin\delta]\frac{\sin^2\phi}{a}C_4\\
&+\Big[(\sin\delta-\delta\cos\delta)(3\varepsilon_3-2\varepsilon-4)\frac{\sin 2\phi}{8a}-\varepsilon_3\delta^2\sin\delta\frac{\sin 2\phi}{8a}\Big]C_5\\
&+\frac{\sin 2\phi}{8a}[(\varepsilon_3-2\varepsilon)\delta\sin\delta-\varepsilon_3\delta^2\cos\delta]C_6\\
&+wa\,\frac{\sin 2\phi}{2}[(-3\varepsilon_3+\varepsilon+2)\sin\delta+\varepsilon_3\delta\cos\delta\\
&+\delta(2\sigma\sin^2\phi+\varepsilon\cos 2\phi)]+m(1+\sigma)\frac{\sin 2\phi}{2}(\delta\cos\delta-\sin\delta)=0.
\end{aligned}
$$

(f)
$$
\begin{aligned}
&-\varepsilon\sin^2\phi(\tan^2\phi\sin\delta+\delta\cos\delta)C_1+\Big[-\frac{\delta\sin\delta}{2}(\varepsilon+\cos^2\phi)\\
&-(\varepsilon+1)\frac{\delta^2\cos\delta}{2}\cos^2\phi+\frac{\varepsilon_3}{12}\delta^3\sin\delta\cos^2\phi\Big]C_2\\
&+\Big[\frac{\sin\delta}{4}(\cos^2\phi-\varepsilon-\varepsilon_1\sin^2\phi)+\frac{\delta\cos\delta}{4}(-\cos^2\phi-3\varepsilon+\varepsilon_1\sin^2\phi)\\
&+(\varepsilon+1)\frac{\delta^2\sin\delta}{2}\cos^2\phi+\frac{\varepsilon_3}{12}\delta^3\cos\delta\cos^2\phi\Big]C_3\\
&+\frac{\tan\phi}{a}\sin^2\phi[\varepsilon_1\sin\delta+(\varepsilon-\varepsilon_1)\delta\cos\delta]C_4\\
&+\frac{\sin 2\phi}{8a}[(\varepsilon_3-2\varepsilon-4)\delta\sin\delta-\varepsilon_3\delta^2\cos\delta]C_5\\
&+\frac{\sin 2\phi}{8a}[(\varepsilon_3-2\varepsilon)\sin\delta-(\varepsilon_3+2\varepsilon)\delta\cos\delta+\varepsilon_3\delta^2\sin\delta]C_6\\
&+\frac{wa}{2}\sin 2\phi.[(2\varepsilon_3-\varepsilon-2)(1-\cos\delta)-\varepsilon_3\delta\sin\delta]\\
&-m\frac{\sin 2\phi}{2}(1+\sigma)\delta\sin\delta=0.
\end{aligned}
$$

$$\left.\begin{aligned}
&\text{(a)}\ C_1 + C_3 = -C_1 - C_2 \sin\delta - C_3\cos\delta - aw\delta\cot\phi.\\
&\text{(b)}\ C_2 = C_2\cos\delta - C_3\sin\delta.\\
&\text{(c)}\ -\cot\phi C_3 + \tan\phi C_1 = \cot\phi(C_2\sin\delta + C_3\cos\delta) - \tan\phi C_1 - aw\delta.\\
&\text{(d)}\ C_4 + C_6 = -C_4 - C_5\sin\delta - C_6\cos\delta\\
&\qquad\qquad - a\delta\cot\phi(C_2\cos\delta - C_3\sin\delta) - wa^2\delta.\\
&\text{(e)}\ C_5 + a\cot\phi C_2 + wa^2 + ma = C_5\cos\delta - C_6\sin\delta\\
&\qquad + a\cot\phi\{C_2(\cos\delta - \delta\sin\delta) - C_3(\sin\delta + \delta\cos\delta)\} + wa^2 + ma.\\
&\text{(f)}\ -\cot\phi C_6 + \tan\phi C_4 - \frac{a}{\sin^2\phi}C_3 - \frac{a}{\cos^2\phi}C_1 = \cot\phi(C_5\sin\delta + C_6\cos\delta)\\
&\qquad - \tan\phi C_4 - a\delta\cot^2\phi(-C_2\cos\delta + C_3\sin\delta)\\
&\qquad + \frac{a}{\sin^2\phi}(C_2\sin\delta + C_3\cos\delta) + \frac{a}{\cos^2\phi}C_1 + wa^2\delta\cot\phi.
\end{aligned}\right\}\quad . \ (54)$$

The remaining two constants are determined by inserting the values from (55) into (52e) and (52f), as these satisfy all the conditions (48). Then

$$d_1C_5 + e_1C_6 = f_1. \qquad d_2C_5 + e_2C_6 = f_2 \quad . \quad . \quad . \quad (56)$$

in which

$$d_1 = \frac{\sin^2\delta}{2}\varepsilon_1\sin^2\phi - \frac{\delta\sin 2\delta}{8}(\cos^2\phi - \varepsilon + \varepsilon_1\sin^2\phi) - \frac{\delta^2}{4}\cos^2\phi(\varepsilon_3 - 2\varepsilon - 2)$$
$$+ \frac{\delta\sin\delta}{2}\sin^2\phi.(\varepsilon_1 - 2\varepsilon) - \frac{\delta^2\cos\delta}{2}\cos^2\phi.(\varepsilon_3 - 1) - \frac{\delta^3\sin\delta}{6}\varepsilon_3\cos^2\phi.$$

$$e_1 = (\sin\delta - \delta\cos\delta)(1 + \cos\delta)\frac{\varepsilon_1\sin^2\phi}{2} + \frac{\delta^2\sin\delta}{2}\cos^2\phi$$
$$+ \frac{\delta\sin^2\delta}{4}(\cos^2\phi - \varepsilon - \varepsilon_1\sin^2\phi) + \frac{\delta^3\varepsilon_3\cos^2\phi}{12}(1 - 2\cos\delta).$$

$$f_1 = -wa^2\delta\cos^2\phi\left[\delta\cos\delta\left(2\varepsilon_3 - \frac{\varepsilon}{2} - 1\right) + \delta\left(\varepsilon_3 - \frac{\varepsilon}{2} - 1\right) + \frac{\delta^2\sin\delta}{2}(\varepsilon_3 - 1)\right.$$
$$\left. + \sin\delta(-3\varepsilon_3 + \varepsilon + 2)\right] - ma\,\delta\cos^2\phi.(1 + \sigma)(\delta\cos\delta - \sin\delta).$$

$$d_2 = \delta^2\sin\delta\frac{\cos^2\phi}{4}(\varepsilon_3 - 2) - \frac{\delta^3\varepsilon_3\cos^2\phi}{12}(1 + 2\cos\delta)$$
$$+ (\cos^2\phi - \varepsilon)\left(\frac{1 - \cos\delta}{4}\right)(\delta\cos\delta + \sin\delta)$$
$$+ \varepsilon_1\sin^2\phi\left(\frac{1 - \cos\delta}{4}\right)(\delta\cos\delta - \sin\delta) + \delta(1 - \cos\delta)\left(\frac{\varepsilon + \cos^2\phi}{2}\right).$$

$$e_2 = \frac{\delta^2\cos^2\phi}{2}(\varepsilon + 1) - \frac{\delta^2\cos\delta}{4}\cos^2\phi.(\varepsilon_3 - 2)$$
$$+ \frac{\delta\sin\delta}{4}(2\varepsilon\sin^2\phi + 2\cos^2\phi + \varepsilon_3\cos^2\phi) + (\cos^2\phi - \varepsilon)\frac{\sin\delta}{4}(\delta\cos\delta + \sin\delta)$$

$$+ \varepsilon_1 \sin^2 \phi \frac{\sin \delta}{4}(\delta \cos \delta - \sin \delta) + \varepsilon_3 \delta^3 \sin \delta \frac{\cos^2 \phi}{6}.$$

$$f_2 = wa^2 \delta \cos^2 \phi \left[\frac{\delta \sin \delta}{2}(3\varepsilon_3 - \varepsilon - 1) - \frac{\delta^2 \cos \delta}{2}(\varepsilon_3 - 1) - (1 - \cos \delta)(2\varepsilon_3 - \varepsilon - 2)\right] + ma \cos^2 \phi . (1 + \sigma)\delta^2 \sin \delta.$$

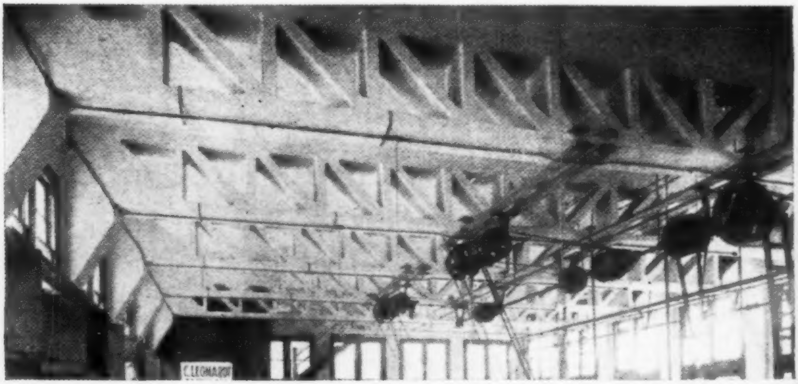
Hence $$C_5 = \frac{f_1 e_2 - f_2 e_1}{d_1 e_2 - d_2 e_1}; \quad C_6 = \frac{d_1 f_2 - d_2 f_1}{d_1 e_2 - d_2 e_1} \quad . \quad . \quad . \quad (57)$$

Equations (55) and (57) give the values of the six constants for a helical beam with both ends fixed, with no more work than in the case when both ends are simply supported. When δ is a multiple of $\frac{\pi}{2}$, $\sin \delta$ or $\cos \delta = 0$ and $\cos \delta$ or $\sin \delta = 1$, and equations (55), (56), and (57) are simplified considerably.

(*To be continued.*)

FIFTY YEARS AGO.

From "Concrete and Constructional Engineering", July–August, 1909.*



Reinforced Concrete Trusses.—The special features (of a laundry building at Los Angeles) are the sixteen trusses used to carry the second floor and the roof. These beams were moulded flat on the ground and then lifted into their permanent positions. All the beams are 37 in. high, those supporting the second floor are 18 in. wide, while those supporting the roof are 14 in. wide, and all are 42 ft. long. The beams are reinforced with square twisted bars. The second-floor beams weigh 19,800 lb., and the roof girders 14,600 lb. The first beam when lifted into place had been cast for one week only, and the last two were lifted after being cast only three-and-a-half days.

* "Concrete and Constructional Engineering" appeared in alternate months until September, 1909.

Flexible Joint for Concrete Pipes.

A FLEXIBLE joint for spigot-and-socket concrete pipes of diameters up to 6 ft. has been introduced recently into Great Britain, and comprises a ring of natural rubber fixed in the socket (*Fig.* 1). The ring has three or four flexible fins, the number depending on the size of the pipe. The manner in which the fins deform when the spigot end of the adjacent pipe is pushed into the socket is shown in *Fig.* 2. It is claimed that the joint permits change of alignment between two connected pipes up to 9 deg. without leakage under a head of water of 40 ft., and that the speed of laying and jointing pipes is much increased. The rubber complies with British Standard No. 2494 and the pipes are made in accordance with B.S. No. 556, except that the tolerances in the dimensions are less in order to ensure a tight-fitting seal.

In a recent test (*Fig.* 3) two pipes 6 ft. 6 in. long and 1 ft. diameter were joined and laid on timber bearers. Concrete plugs were inserted in the open ends and the pipes were filled with water. The pressure of the water was increased to 20 lb. per square inch and maintained at this pressure throughout the test. The ends of the pipes were raised off the end bearers by jacks, the weight of the pipe and water being sufficient to keep the pipes on the inner bearers. When leaking occurred the pipes were $6\frac{1}{4}$ in. out of alignment (*Fig.* 3), the angle between the pipes was $9\frac{3}{4}$ deg., and the spigot was drawn out of the socket a distance of $1\frac{5}{8}$ in. A slight relaxation of the deformation caused the leak to cease.

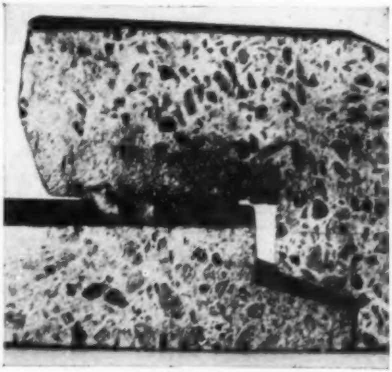


Fig. 2.

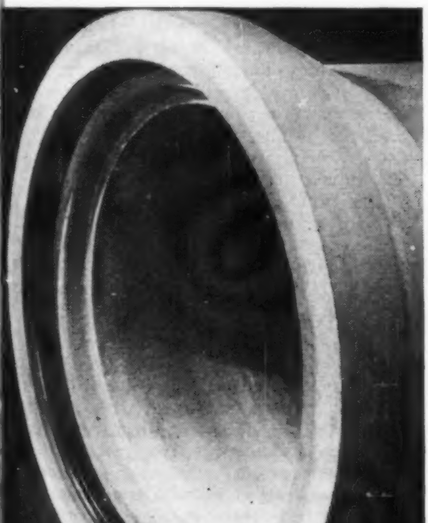


Fig. 1.—The Rubber Water-seal.

In another test two pipes similar to those in the foregoing were joined together and a watertight metal and rubber annular box was placed around the outside of the joint. The box was filled with water which was maintained at a pressure of 20 lb. per square inch. The pipes were then forced out of alignment as in the first test, and the deformation was continued until water leaked through the joint into the pipe. Leaking was first detected when the pipes were 5 in. out of alignment ($8\frac{1}{4}$ deg.) and the spigot was drawn out of the socket a distance of $1\frac{3}{8}$ in.; the leak ceased when the deformation was relaxed slightly.

Fixing the Rubber Strip.

The inner face of the socket is roughened by brushing immediately after casting the pipe. When the pipes have hardened they are placed vertically with the socket

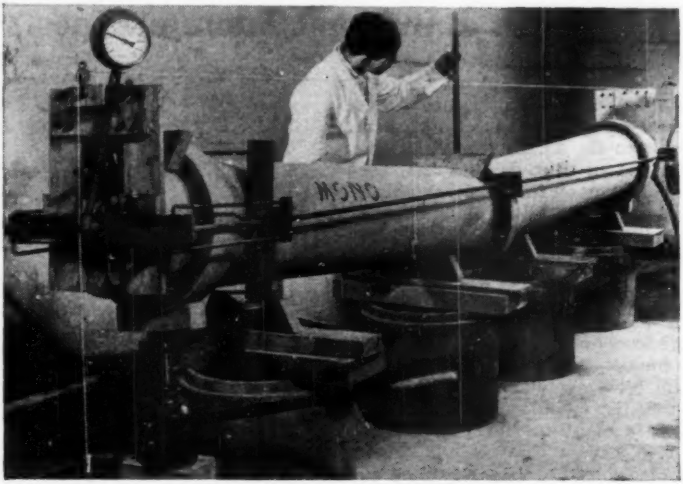


Fig. 3.—Test in Progress.

uppermost. The rubber strip, which has been cut and vulcanised to form a ring to suit the dimensions of the socket, is placed around a metal former, which is placed inside the socket. After the concrete has been wetted, the space between the rubber and socket is packed with high-alumina cement mortar. Grooves in the outer face of the rubber ensure a key between it and the mortar. The metal former is vibrated to compact the mortar, which is then smoothed with a trowel. The former is removed when the mortar has hardened.

Laying Pipes.

The tackle required for laying pipes with these joints comprises a wire rope, one end of which is wrapped around behind the socket of a pipe already laid, and a straining gear through which the other end of the rope is passed. A hook on the straining gear is attached to the socket of the pipe being laid and the spigot of this pipe is forced into the socket of the pipe laid previously. In a recent test six pipes were laid and jointed in less than ten minutes. Before the joint is made the spigots are painted with soap-jelly lubricant.

This joint (the Tylox), which has been used for many years in the U.S.A. and on the Continent, is now fitted to concrete pipes in Great Britain by the Mono Concrete Co., Ltd., who also provide the tackle for laying the pipes.

Reaction between Aggregates and Cement.

THE discovery in the U.S.A. a few years ago of failures of concrete due to the reaction between the alkalis in cement and in some aggregates seems to have started research on this subject in many countries, and voluminous reports, generally recounting similar tests and reaching the same conclusions, have been published in the U.S.A., Denmark, Sweden, Australia, Great Britain, and elsewhere. Two more reports on this subject are Research Papers Nos. 20 and 25 of the Department of Scientific and Industrial Research (price 4*s.* and 4*s.* 6*d.* respectively). The tests described are said to confirm what was already known, namely that the aggregates in common use in this country are not reactive at normal temperatures even with cements with high-alkali content. Other reports recently issued on this subject are the thirty-fifth and thirty-sixth reports on "Studies in Cement Aggregate Reaction" of the Scientific and Industrial Research Organisation of Australia, where damaged concrete has resulted from the use of volcanic rocks.

Effects of Mining Subsidence.

FORMS of subsidence of the ground due to mining and the effects of such movements on the stability of structures and other works on the surface are dealt with in a report* issued by the Institution of Civil Engineers. Some recommendations are given for remedial measures for damaged structures, and precautions that can be taken when designing and constructing new structures are described. The following notes are abstracted from the report.

Damage can be caused to structures by general subsidence, tilting, and tensile and compressive horizontal strains, and is generally caused by the differential horizontal movement of the ground surface. Convexity of the ground causes surface extension which tends to produce tension in foundations, walls, and roofs, thus producing a gap tapering from the ground upwards. Concavity of the ground causes compression with a tendency to crush foundations, walls, and roofs. When damage from compressive strain occurs it tends to be severe. As a precautionary measure no structure should be erected within 50 ft. of the surface position of a known fault.

Buildings.

In general statically-indeterminate structures such as fixed arches and rigid frames, which are sensitive to relative movements of supports either vertically or horizontally, should be avoided. Freely-supported structures braced and tied at the roof, floor, and foundation levels are recommended. Buildings should be so placed that the shorter axis coincides with the maximum curvature of the ground due to subsidence.

Where considerable ground movements are expected structures should be built on reinforced concrete rafts, the design of which should be such that horizontal strains in the ground are not transmitted to the raft. Prefabricated buildings erected on thin concrete rafts will conform to most movements.

Small buildings should be erected on a reinforced concrete slab 6 in. thick, which should have no projections on the underside and should be on a base of sand or other granular material about 6 in. thick and covered with waterproof paper. The total tensile strength of the slab in the direction of either principal axis should be sufficient to resist a force equal to one-third of the weight of the structure, assuming that the permissible tensile stress in mild steel reinforcement is 30,000 lb. per square inch and that the permissible compressive stress in 1 : 2 : 4 concrete is 2000 lb. per square inch.

If reinforced concrete strip footings are provided and laid on fine granular material a gap should be left between the faces of the excavation and the footing and filled with compressible material.

Large buildings several stories high in mining areas should be in separate parts not exceeding about 60 ft. square on plan, with gaps between them 2 in. wide or more according to the height of the building to allow for the movement expected. The foundations should also be independent. Single-story and other light structures may be founded on a reinforced concrete slab 6 in. to 15 in. thick on a 6-in. layer of granular material. For heavier and multiple-story buildings a cellular raft or a beam-type substructure may be provided on a layer of granular material. Where shallow or uncharted workings occur and other seams of coal are to be worked a cellular raft is recommended. If the amount of movement can be predicted with reasonable accuracy the raft and frame of the building should be designed to accommodate deflections up to the values given by Skempton and Macdonald.(1)

Important buildings should be provided with jacks under the main supports to allow vertical adjustment. Horizontal movement should be provided for by separating the columns from their foundations by a sliding joint. In the case of a heavy building which covers a large area and cannot have a rigid continuous foundation one column may be provided as a self-supporting bastion to which all other members are attached by flexible hinges. The column may produce a stiff corner at high level instead of transmitting moment to the ground. The hinge system is complex, and vertical and horizontal jacks are necessary at the column bases to control distortion. Structures of small area such as platforms for plant or machines can be supported on three-point spherical bearings.

* "Report on Mining Subsidence." (London: Institution of Civil Engineers. 1959. Price 10s.)

Measures for remedying the effect of settlement are often obvious and depend on local conditions. It is important to recognise the type of damage and cause of the fracture. The type of repair should be related to the time of repair. For example, if a building is damaged through extension it must be allowed to contract and fractures must be kept open in order that they may be free to close when the advance of the workings causes compression. Permanent repairs should be made when the workings have advanced beyond the critical area and movement has ceased, provided that no further workings are expected.

Bridges.

Bridges on sites subject to movement may be (a) flexible, that is jointed or freely-supported structures that can distort and, if necessary, be jacked back into position after movement; or (b) rigid, that is designed as rigid frames that can resist the strains imposed by movement, but this type would normally be used only for small bridges and culverts. If the deck of a bridge has to be maintained at a specified level, a flexible structure is generally preferable. Rubber bearings provide a useful means of accommodating lateral, longitudinal, and rotational distortion, particularly if combined with jacks to make adjustments for vertical movement.

The design of bridges should allow for the greatest movements likely, and the following factors should be considered. Bridges with freely-supported decks are preferable. Statically-indeterminate structures should generally be avoided. Cantilevered and suspended spans are suitable but the bearings for suspended spans must allow horizontal movement in any direction. Open abutments which allow the embankment to spill through are preferable to solid abutments and wing-walls, but if abutments and wing-walls are used they should be designed separately with provision for independent movement. The joint between an abutment and a wing-wall should be such that movement of the wall does not affect the abutment. Alternatively the wing-walls, if they are not too large, may be cantilevered from the abutment. The increase in earth pressure developed by horizontal surface compression behind the abutment should be taken into account. Where provision is made for lifting the deck, the abutments and wing-walls should be designed for the final conditions of height and earth pressure so that they can be extended when required.

If the deck is sloping the lower end should be fixed and all expansion allowed for at the higher end, and it may be necessary to provide extra strength at the lower end where additional forces may occur due to movement.

Reservoirs.

Most older reservoirs and storage tanks are constructed, according to the practice of the time, of plain concrete or brickwork with puddled clay under the floors and behind the walls. This form of construction has proved generally effective in counteracting the effects of movement, but this may be because most of such reservoirs and tanks are relatively small. This construction still has its advocates, but a monolithic structure of reinforced concrete with columns and with either a beam-and-slab or a flat-slab roof is now more general. Expansion and contraction joints are seldom provided and the roof and floor are continuous with the walls. Small cracks caused by changes of temperature are sealed by ironing-in bitumen. Copper or rubber seals are normally used to ensure watertightness at the construction joints of the walls. Variations of this practice are to provide freedom of movement between the walls, roof, and floor, and to ensure that cracks due to subsidence will occur along predetermined planes.

In general reservoirs and tanks should be small, that is with capacities of from 250,000 to 500,000 gallons, connected, if necessary, by flexible-joint piping. For larger capacities one reservoir with independent sub-divisions is often preferred.

It is uneconomical to design a reservoir to resist the stresses which are likely to arise as a result of subsidence because these are indeterminate and, subject to the foregoing, it is unnecessary to depart from the Code of Practice for reinforced concrete structures containing liquids.

(1) "Allowable Settlements of Buildings", by A. W. Skempton and D. H. Macdonald, Proc. Inst. Civil Engrs. Pt. III, December, 1956.

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